**Exchange-of-Thought: Enhancing Large Language Model Capabilities through Cross-Model Communication**

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# Abstract



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Review

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Large Language Models (LLMs) have recently made significant strides in complex reasoning tasks through the Chain-of-Thought technique. Despite this progress, their reasoning is often constrained by their intrinsic understanding, lacking external insights. To address this, we propose Exchange-of-Thought (EoT), a novel framework that enables cross-model communi- cation during problem-solving. Drawing inspi- ration from network topology, EoT integrates

four unique communication paradigms: Mem- ory, Report, Relay, and Debate. This paper

1. CoT
2. Self-Correction
3. EoT

delves into the communication dynamics and volume associated with each paradigm. To counterbalance the risks of incorrect reason- ing chains, we implement a robust confidence evaluation mechanism within these communi- cations. Our experiments across diverse com- plex reasoning tasks demonstrate that EoT sig- nificantly surpasses established baselines, un- derscoring the value of external insights in en- hancing LLM performance. Furthermore, we show that EoT achieves these superior results in a cost-effective manner, marking a promis- ing advancement for efficient and collaborative AI problem-solving.

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“Two heads are better than one.”

–*English Proverb*

# Introduction

Large Language Models (LLMs) such as GPT- 4 (OpenAI, 2023) are revolutionizing the field of Natural Language Processing (NLP) by utilizing vast training corpora and huge computational re- sources (Bai et al., 2022a; Ouyang et al., 2022; Chowdhery et al., 2022; Zhang et al., 2022; Tou- vron et al., 2023a, *inter alia*). Although LLMs achieve exemplary performance across a wide range of NLP tasks (Wei et al., 2022a; Chung et al., 2022), they consistently struggle to perform well in

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Figure 1: Comparison of CoT, Self-Correction, and EoT. Both CoT and Self-Correction rely on the model’s innate abilities to generate and refine output, lacking external insights. EoT enhances the model’s reasoning ability by incorporating the thoughts of other models as external insights.

reasoning tasks, and this limitation cannot be over- come solely by increasing the size of models (Rae et al., 2022; bench authors, 2023).

To overcome this shortcoming, Wei et al. (2022b) proposed chain-of-thought (CoT) prompt- ing, which guides the model to generate a series of intermediate reasoning steps before reaching the final answer. At the same time, a series of self-correction methods (Welleck et al., 2023; Gan- guli et al., 2023) have been proposed, which aim to iteratively improve the quality of answers by leveraging the model’s feedback to their previous outputs (Madaan et al., 2023; Shinn et al., 2023).

However, CoT and self-correction solely base on the model’s own understanding and perspective of the question during the reasoning process. Recent studies (Huang et al., 2023; Valmeekam et al., 2023; Stechly et al., 2023) indicate that LLMs struggle to revise their responses without external feedback. This can be attributed to the model’s complete de- pendence on internal representations to generate responses, which makes it difficult to overcome inherent limitations in capability (Yin et al., 2023).

100



Contain correct

Not contain correct

86

79

57

59

54

23

80

60

Quantity

40

20

0

GSM8K AQuA SVAMP

solving process with a diversity of insights. Fur- thermore, we delve into the intricacies of each com- munication paradigm, analyzing the dynamics of information flow and the volume of communica- tion. With the awareness that both correct and incorrect reasoning chains propagate within com- munications, we introduce confidence evaluation mechanisms that employs the analysis of answer variations to assess models’ confidence levels. It is designed to mitigate the influence of erroneous reasoning, thereby ensuring the integrity and relia-

Figure 2: Pilot experiments on three reasoning datasets. The number of erroneous samples containing the correct answer is significantly higher than those not containing the correct answer.

Despite the undeniable importance of external insights (Yao et al., 2023), acquiring high-quality external insights remains a challenge. Wang et al. (2023c)’s research suggests that the single reason- ing chain generated by CoT limits the model’s rea- soning performance. By increasing the temperature to sample diverse reasoning chains and selecting answers through majority voting, the model’s rea- soning performance can be further improved. How- ever, when confronted with difficult questions, the model often yields a higher number of incorrect responses. In Figure 2, our analysis of correct and incorrect answers within erroneous samples from three reasoning datasets reveals that in most cases the model can deduce the correct answer.

In human society, the truth, even when held by a minority, can gain widespread acceptance and recognition through clear and persuasive commu- nication (Le Bon, 1897). The correct reasoning of others can serve as high-quality external in- sights, enriching and elevating our collective under- standing. Thus, we propose Exchange-of-Thought (EoT), a novel framework that fosters cross-model communication during the problem-solving pro- cess. This initiative enables models to incorporate the reasoning of others as external insights.

Figure 1 contrasts EoT with CoT and self- correction methods, highlighting the unique ap- proach of EoT in integrating external perspec- tives. Inspired by the principles of network topol- ogy (Bisht and Singh, 2015) and agent communi- cation (Parsons and McBurney, 2003), we propose four communication paradigms: Memory, Report, Relay, and Debate. These paradigms are designed to facilitate the exchange of ideas and reason- ing chains among models, enriching the problem-

bility of the problem-solving process.

Experiments across various complex reasoning tasks demonstrate that EoT significantly outper- forms established strong baselines, underscoring the critical role of external insights in augment- ing the capabilities of LLMs. We summarize our contributions as follows:

* We introduce Exchange-of-Thought (EoT), a pioneering framework for cross-model com- munication that incorporates external insights from other LLMs during problem-solving.
* We present and examine four communication paradigms coupled with a confidence evalua- tion mechanism that assesses model certainty through the variability of answers, mitigating the impact of incorrect reasoning.
* Experimental results on various complex rea- soning tasks underscore the efficacy and cost- effectiveness of EoT, highlighting the signif- icance of incorporating external insights and communication in problem-solving.

# Related Work

## Chain-of-Thought prompting in LLMs

Wei et al. (2022b) highlight that LLMs can man- ifest enhanced reasoning capabilities when being prompted by demonstrations with intermediate rea- soning steps. This technique can effectively im- prove the performance of LLMs on complex rea- soning tasks (Wei et al., 2022a; Kojima et al., 2022). A series of strategies for enhancing CoT has been proposed to further improve the performance of LLMs. One such method is program-aided lan- guage models (Gao et al., 2022; Chen et al., 2022), which aims to decouple reasoning and computa- tion through program synthesis. Moreover, com- plex tasks can also be transformed into delegable sub-tasks through modular approaches (Khot et al., 2023). Choosing appropriate demonstrations can

also enhance the performance of CoT (Li et al., 2023a; Li and Qiu, 2023a). Notable among these, AutoCoT (Zhang et al., 2023b) uses an automated way to construct and sample diverse demonstra- tions. Active-Prompt (Diao et al., 2023) selects the most helpful samples for labeling based on the model’s uncertainty in the outputs. Recently, Li and Qiu (2023b) employ a strategy of storing high-confidence thoughts as external memory and retrieves these insights to aid the reasoning process.

## Ensemble of Reasoning Paths

LLMs have the ability to explore multiple rea- soning paths using techniques such as tempera- ture adjustment and prompt sampling (Chu et al., 2023). Wang et al. (2023c) suggest that for com- plex questions, there may be several correct paths to approach a problem, leading to the proposal of Self-Consistency. This method replaces the greedy decoding strategy with the sampling of multiple rea- soning paths and selecting the most consistent an- swer, resulting in significant performance improve- ments. Beyond that, Fu et al. (2023b) discover that prompts with higher reasoning complexity could achieve better performance in multi-step reason- ing tasks, leading to the proposal of complexity-

chains and action plans. For numerical problems, Zheng et al. (2023) gradually guide models to the correct answer by using previously generated an- swers as hints. With the aid of external knowledge, Wang et al. (2023a) introduce chain-of-knowledge prompting that employs evidence triples to curb the generation of unfactual and unfaithful answers. Taking model interactions into account, multi-agent debates (Du et al., 2023; Liang et al., 2023) have been introduced to enhance the factual accuracy of generated content and reduce fallacies and hal- lucinations. EoT differs from these efforts as we prioritize enhancing the current reasoning process generated by a single model by incorporating the reasoning processes from other models as external insights through cross-model communication.

# Preliminary

Firstly, we define the current methods that use LLMs to solve problems. We denote a LLM with a parameter size of *θ* as *pθ*, and the sequence length as *t*, which includes tokens [*s*1*, s*2*, . . . , st*]. The LLM predicts the next token based on the prior tokens in the sequence. The probability of the *si* token is *pθ*(*si*∣*s*1*, s*2*, . . . , si*−1). Therefore,

the probability of the whole sentence is *pθ*(*s*) =

∏

based prompting. While other methods, such as re-ranking (Cobbe et al., 2021; Thoppilan et al.,

*t i*=1

*pθ*(*si*∣*s*≤*i*−1).

2022), have also been applied to select suitable rea- soning paths, they often rely on heuristic or trained smaller models. Recently, Li et al. (2023b) sam- ple different demonstrations and use step-by-step verification to filter out incorrect answers. How- ever, obtaining step-level labels can be challenging, and using smaller models for judgment struggles to handle complex reasoning processes. In con- trast, our method fully utilizes the communication and decision-making capabilities of LLMs to reach the final answer, without the need for additional training and annotated data.

## 2.3 Reasoning Path Refinement

Although CoT (Wei et al., 2022b) effectively en- hances the performance of LLMs in complex rea- soning tasks, they remain susceptible to errors dur- ing the reasoning process, leading to incorrect an- swers (Bai et al., 2022b; Lyu et al., 2023). To mitigate this issue, starting from the model’s own thoughts, Shinn et al. (2023) and Madaan et al. (2023) employ the model’s own feedbacks and past mistakes to refine the reasoning process. Yao et al. (2023) explore the synergies between reasoning

**Standard prompting.** Standard prompting in- volves deriving an answer *a* from a question *q* using *pθ*(*a*∣*q*). In-Context Learning (Brown et al., 2020)

aims to improve LLMs performance by adding demonstrations *D* = {*d*1*, d*2*, . . . , dn*} to the input, which can be expressed as *pθ*(*a*∣*D, q*).

**CoT prompting.** As identified by Wei et al. (2022b), the incorporation of intermediate reason- ing steps can improve the proficiency of LLMs in tackling complex reasoning challenges. To facili- tate this, a rationale *ri* is added to demonstration

*di* = {*qi, ri, ai*} to guide the LLMs in explicitly

generating reasoning steps. Fu et al. (2023b) ob-

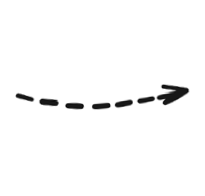
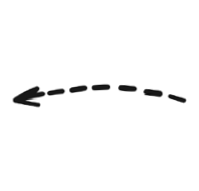
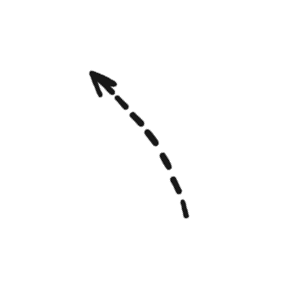
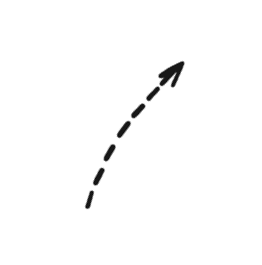
serve that using rationale *ri* with more complex reasoning steps for demonstrations can further en- hance the model’s reasoning performance.

**Self-Consistency.** Self-Consistency method, in- troduced by Wang et al. (2023c), effectively con- solidates answers from multiple independent rea- soning chains. This technique prioritizes the most commonly occurring answer, defined as

*a* = argmax*aif* (*ai*), where *f* (*ai*) denotes the fre-

quency of each answer *ai*. This approach enables

A A A A

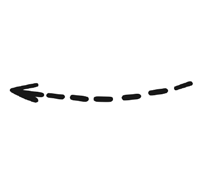
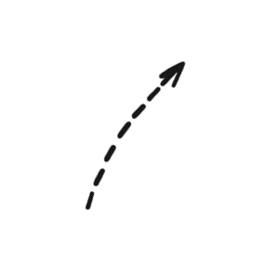
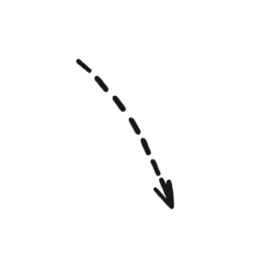


Tree

A

B

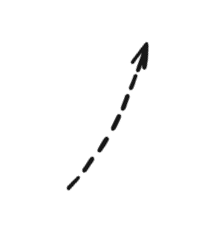
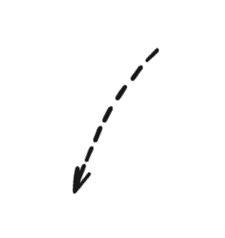
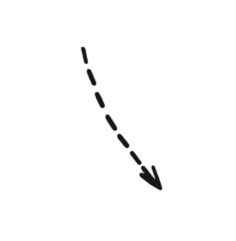
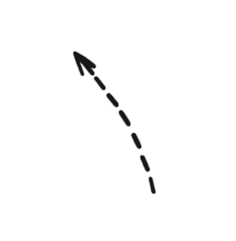
C



A

B

C



A

B

C

B

Bus

B

Star

B

Ring

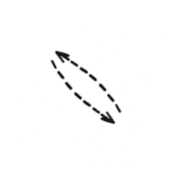
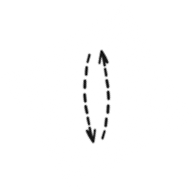
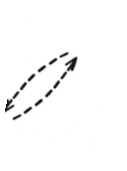
B

C

C

C

C



A

B

C

Memory

Report

Relay

Debate



A B C

A

B C



A B C

A

B C



A B C

A

B C



A B C

A

B C

Fully Visible

Central Visible

Neighbor Visible

Peers Visible

Figure 3: Correspondence between communication paradigms and network topologies. The top row depicts four network topologies. The second row correlates these with the corresponding communication paradigms. The bottom row offers an analysis of the communication volume associated with each paradigm. The horizontal axis represents the information that the node can receive, while the vertical axis indicates the information that the node can send.

the model to explore a broader range of reasoning pathways, thereby enhancing its reasoning ability. However, it remains constrained by the intrinsic limitations of LLMs’ capabilities.

**Progressive-Hint Prompting.** Introduced by Zheng et al. (2023), Progressive-Hint Prompting (PHP) leverages a sequence of historical answers

## Communication Paradigm

Inspired by network topology (Bisht and Singh, 2015) and intelligent agent communication (Par- sons and McBurney, 2003), we propose four com- munication paradigms to determine the counter- parts for model communication. As illustrated

{*a*(1)

*, a*(2)

*, . . . , a*

(*j*−1)

} to enhance the current rea-

in Figure 3, we propose Memory, Report, Re-

lay, and Debate communication paradigms each

soning process *r*(*j*) and facilitate the derivation of the subsequent answer *a*(*j*).

# Methodology

We introduce Exchange-of-Thought (EoT), a novel framework designed to facilitate cross-model com-

corresponding to the Bus, Star, Ring, and Tree network topologies, respectively. Assume in *j*- th round of communication, given a set of LLMs

{*M* } = {*m*1*, m*2*, . . . , mn*}, the model *mi* gener-

ates the corresponding rationale *r*(*j*) and the an-

*i*

swer *a*(*j*) based on the (*r*(*j*−1)*, a*(*j*−1)), where *K*

munication, allowing for the exchange of reasoning *i K K*

processes to integrate external insights. This inno- vative approach leverages the communicative abil- ities of LLMs to promote the sharing of rationale *r* and answer *a* among participating models, fos- tering a collaborative environment for thought and analysis. The implementation of EoT encounters three key challenges:

1. How to identify the appropriate counterparts for model communication?
2. What are the conditions for ceasing communi-

is the set from which model *mi* can receive reason- ing processes. In the first round, we use the CoT method proposed by Wei et al. (2022b) to generate

(*r*(1)*, a*(1)) ∼ *Pθ*(*r*(1)*, a*(1)∣*D, q*).

**Memory.** Under the Memory paradigm, all mod- els record their rationale *r* and answer *a* in a log- book, which is fully visible from all models. This means that in the *j*-th round, any model, such as model *mA*, can access the reasoning chains and an-

swers from all models (*r*(*j*−1)*, a*(*j*−1))*, m* ∈ {*M* }.

cation between models? *m m*



1. How to minimize the influence of incorrect reasoning during the communication process?

As depicted in Figure 3, this paradigm facilitates the fastest flow of information and also incurs the highest communication cost among all paradigms.

**Report.** Under the Report paradigm, we desig- nate model *mA* as the central node, which can ob- tain the rationale and answer from all other models

(*r*(*j*−1)*, a*(*j*−1))*, m* ∈ {*M* }\{*mA*}. Both *mB* and

information from the central node. In addition, each node can receive information from its previ- ous round. Thus, the total communication volume is 3*n*−2. The transmission from a non-central node

*m m*

*mC* only receive information from *mA* and do not interact with each other. Consequently, *mA* plays a pivotal role in the communication process. This paradigm also allows for rapid information flow, but it demands a higher capacity for processing and analysis for the central node.

**Relay.** Under the Relay paradigm, we order the models by number and connect them in a circle. Each node is capable of receiving information from the preceding node and transmitting its own infor- mation to the subsequent node. For example, in the

*j*-th round, *m* passes (*r*(*j*−1)*, a*(*j*−1)) to *m* and

*A A A C*

receives (*r*(*j*−1)*, a*(*j*−1)) from the previous round

to another non-central node requires two transmis- sions, whereas sending to the central node requires only one. Thus, the average communication vol- ume is calculated as 2 − 2 .

Under the Relay paradigm, each node receives information from the preceding node and its own information from the last round, resulting in a com- munication volume of 2*n*. Node *i* sends informa- tion to node *i* + 1 in just one transmission, but sending to node *i* − 1 requires *n* − 1 transmissions. Therefore, the average propagation speed is *n* .

*n*

2

In the Debate paradigm, nodes are assumed to form a full binary tree with a height of *h* =

⌈log (*n* + 1)⌉. The communication volume for

*B B* 2

of *mB*. This distributed communication mode can reduce the demands on the information processing capacity of each node, but it may result in a slower flow of information.

**Debate.** We have adapted the tree topology to devise the Debate paradigm. This paradigm per- mits leaf nodes to exchange information with each other, while parent nodes are solely responsible for aggregating information, meaning that informa- tion flow is directed upward from child to parent. As illustrated in Figure 3, *mB* and *mC*, as child

each pair of child nodes is 4, and it is 3 for the parent node. Consequently, a subtree comprising two children and one parent has a communication volume of 7. The number of non-leaf nodes in a full binary tree is *n*−1 , leading to a total communication

volume of 7(*n*−1) . Information under the same par- ent node requires only one transmission, whereas the information from the farthest nodes needs *h* − 1 transmissions to converge at the root node. Thus,

2

2

the communication speed *S* = Σ*h*−12*i*−1*i* .

*i*=1

2*h*−1−1

nodes, are able to communicate, whereas *mA*, as a parent node, can only receive information from its children. This communication paradigm strikes a balance between the model’s information process- ing capacity and the speed of information flow.

## Communication Volume

The last row of figure 3 displays the information that can be transmitted and received in different communication paradigms. The communication volume is measured by the number of messages

## Termination Condition

Utilizing the models’ current round outputs and the answers from previous rounds, we have devised two criteria for terminating communication: con- sistent output and majority consensus.

**Consistent Output Termination.** Inspired by Zheng et al. (2023), we implement a consistent output termination in EoT. The termination con- dition is triggered when the output of model *mi* in the *j*-th round is the same as the output in the

*j* − 1-th round, *a*(*j*) = *a*(*j*−1). In this case, *m* will

received, assuming there are *n* models participating in the communication, with each node transmitting its information from the previous round to the next. In the Memory paradigm, every node receives information from all other nodes in the previous round, resulting in a communication volume of *n*2. Any piece of information requires only one

transmission to reach the corresponding node.

Under the Report paradigm, the central node re- ceives information from *n* − 1 non-central nodes, while each of the *n* − 1 non-central nodes receives

*i i* *i*

stop receiving or sending information and exit the current communication.

**Majority Consensus Termination.** Du et al. (2023) observed that LLMs can converge on a con- sensus after several rounds of debate, suggesting that LLMs fine-tuned with reinforcement learn- ing from human feedback (RLHF) (Ouyang et al., 2022) are more likely to reach an agreement. In- spired by this finding, we propose the termination condition of majority rule, where LLMs cease com-

|  |
| --- |
| **Turn 1:** 20 dozen cups are equivalent to 20 x 12 = 240 cups… The answer is $70. |
| **Turn 2:** One dozen is equal to 12, so 20 dozen cups is 20 x 12 = 240 cups…A's solution is accurate. So, the total cost is $145. |
| **Turn3:** Let's first convert 20 dozen cups to a total number of cups: 20 x 12 = 240 cups, …Therefore, the total cost of buying each cup is 90 dollars. |

|  |
| --- |
| **Turn 1:** First, we need to convert 20 dozen cups to the total number of cups…Dividing both sides by 240: x = $70 Therefore, the total cost is $70. |
| **Turn 2:** Both A and B have provided their solutions.…Dividing both sides by 240: c = $70. So, the cost of buying each cup is $70. |
| **Turn3:** Based on the solutions provided by A and B… x = $70 So each cup costs $70. |

Figure 4: An illustrative comparison between a confident model and an unconfident model. Model A generates three different answers over three communication rounds, indicating uncertainty about the answer, while Model B consistently adheres to a single answer.



B

Low Confidence



High Confidence



A

munication with each other once a majority of them reach an agreement. This approach serves as a global termination condition, distinguishing it from the consistent output termination, which acts as a cessation criterion on an individual model basis.

## Confidence Evaluation

An intriguing aspect of human behavior is that in- dividuals are less likely to make mistakes when they are confident in their answers. Conversely, when uncertain about their answers, they become more susceptible to the influence of others’ opin- ions. Additionally, as found by Wang et al. (2023c), the likelihood of an answer being correct decreases as the generated results become more contradictory. Therefore, if a model’s answers frequently change during communication, there is a high probability that these answers are incorrect.

We propose calculating the model’s confidence based on the variation in responses. This aids the recipient of the information in verifying its relia- bility, thereby safeguarding the problem-solving process from the disruption of erroneous informa- tion. Figure 4 presents an illustrative example of a confident model and a non-confident model.

In a communication with *k* rounds, model *mi*

generates a set of answers {*a*(1)*, . . . , a*(*k*)}. Let

and Roth, 2015), SingleEQ (Koncel-Kedziorski et al., 2015), AddSub (Hosseini et al., 2014), AQuA (Ling et al., 2017), and SVAMP (Patel et al., 2021). **(2) Commonsense Reasoning**: We utilize the CommonsenseQA(CSQA; Talmor et al., 2019) and StrategyQA (Geva et al., 2021). **(3) Symbolic Reasoning**: We employ two datasets from Big- Bench (bench authors, 2023; Suzgun et al., 2023), namely Penguins in a Table (Penguins) and Date Understanding. In Appendix B, we provide a de- tailed description and statistics of the datasets.

**Baselines.** We compare EoT with a series of strong baselines, which include (1) Chain-of- Thought prompting (CoT; Wei et al., 2022b), (2) Complexity-based prompting (ComplexCoT; Fu et al., 2023b), (3) Self-Consistency (SC; Wang et al., 2023c), (4) Progressive Hint Prompting (PHP; Zheng et al., 2023). Specifically, CoT and ComplexCoT are prompting methods, while SC and PHP are reasoning chain ensemble methods. For simplicity in notation, we use “CoT-SC(10)” to denote the approach that employs the CoT prompt method to sample 10 reasoning chains and then utilize the SC method to select the answer.

**Implementation Details.** We access the GPT models through the OpenAI API. In the main

*i i* experiments, we employ GPT-3.5-Turbo-0301

*f* (*a* ) = max #{*a* ∣ *a* = *a*(*j*)} denote the num- (GPT-3.5) and GPT-4-0314 (GPT-4) to evaluate

*i* *i*

ber of the most frequently occurring answer from model *mi*. Consequently, we obtain the model’s

confidence level *Ci* = *f*(*ai*) in the current round.

*k*

# Experiments

## Experimental Setups

**Tasks and Datasets.** In our experiments, we eval- uated the performance of EoT across three com- plex reasoning tasks: **(1) Mathematical Reason- ing**: This involves six datasets, which includes GSM8K (Cobbe et al., 2021), MultiArith (Roy

the effectiveness of EoT in comparison to other strong baselines. We set the temperature at 1 dur- ing the generation. The prompts for CoT and Com- plexCoT are sourced from Wei et al. (2022b) and Fu et al. (2023b). By default, we employ three GPT-3.5-Turbo-0301 to engage in the EoT com- munication. We apply the majority consensus ter- mination and confidence evaluation, selecting the majority answer as the final outcome. Taking into account the impact of temperature, we report the average performance and standard deviation across five runs. Additionally, in Section 5.3, to further

**Method GSM8K MultiArith SingleEQ AddSub AQuA SVAMP Avg.**

*Single Reasoning Chain*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| CoT | 79.12±0.50 | 97.27±0.65 | 92.80±0.27 86.23±0.52 | | 55.12±1.03 | 79.52±0.81 | 81.67 |
| ComplexCoT | 79.32±0.65 | 95.40±0.50 | 91.34±0.33 84.46±0.86 | | 56.46±0.59 | 77.70±0.54 | 80.78 |
| CoT (GPT-4) | **94.90** | 97.80 | 93.10 **89.30** | | **77.50** | **90.50** | 90.51 |
| *Ensemble Methods* | | | | | | | |
| CoT-SC(3) | 82.82±0.32 | 98.20±0.43 | 93.31±0.12 | 87.19±0.47 | 62.13±1.30 | 81.98±0.49 | 84.27 |
| CoT-SC(5) | 85.47±0.52 | 98.60±0.08 | 93.70±0.25 | 87.49±0.38 | 64.02±0.95 | 83.76±0.81 | 85.50 |
| CoT-SC(10) | 87.57±0.27 | 98.97±0.12 | 94.06±0.36 | 87.59±0.58 | 66.38±1.72 | 84.96±0.33 | 86.59 |
| ComplexCoT-SC(3) | 84.17±0.67 | 97.43±0.31 | 92.95±0.53 | 86.13±0.74 | 60.47±1.55 | 81.44±0.79 | 83.77 |
| ComplexCoT-SC(5) | 87.26±0.33 | 98.13±0.22 | 94.02±0.29 | 86.48±0.61 | 62.05±2.40 | 83.86±0.92 | 85.30 |
| ComplexCoT-SC(10) | 89.23±0.31 | 98.23±0.37 | 94.21±0.16 | 86.58±0.58 | 64.96±1.91 | 85.58±0.87 | 86.46 |
| PHP | 85.10 | 98.00 | 92.90 | 85.30 | 60.60 | 83.10 | 84.16 |
| *Exchange-of-Thought* | | | | | | | |
|  |  |  | | | | |  |
| EoT-Memory | 88.98±0.89 | 98.80±0.16 94.09±0.48 87.65±0.49 69.37±2.77 84.28±0.48 | | | | | 87.20 |
| EoT-Report | 88.61±0.83 | **99.03**±**0.22**  94.06±0.47 | | | | | 87.46 |
| EoT-Relay | 88.42±0.72 98.97±0.16 94.13±0.49 | | | | | | 87.50 |
| EoT-Debate | 88.52±0.76 98.90±0.17 **94.25**±**0.19**  87.70±0.34 69.69±1.24 85.10±0.24 | | | | | | 87.36 |

Table 1: Comparison of EoT performance with a series of strong baselines on mathematical reasoning tasks. The best results are highlighted in bold, while the best results among different EoT paradigms are underlined. The performance of different EoT communication paradigms is represented by varying colors, with darker shades indicating higher performance. The results for CoT (GPT-4) and PHP are reported from Zheng et al. (2023).

|  |  |  |
| --- | --- | --- |
| 87.95±0.34 | 70.31±2.19 | 84.78±0.75 |
| 87.59±0.58 | 70.87±1.98 | 85.04±0.31 |

validate the performance of different LLMs on EoT, we incorporate the Claude-2 model. The further implementation details are listed in Appendix C.

## Performance of EoT

**Mathematical Reasoning.** According to the re- sults presented in Table 1, the four communication paradigms of EoT have shown significant improve- ment over both CoT and ComplexCoT in mathe- matical reasoning tasks. Compared to the currently strongest baseline method, PHP, the Memory, Re- port, Relay, and Debate paradigms have respec- tively increased the average performance by 3.04%, 3.30%, 3.34%, and 3.20%. EoT comprehensively outperforms CoT-SC(5), achieving performance comparable to, or even surpassing, that of CoT- SC(10). When compared to the current best LLM GPT-4, three GPT-3.5 with EoT surpassed a single GPT-4 with CoT on the MultiArith and SingleEQ datasets. This indicates that through cross-model communication and collaboration, three less ca- pable models can compensate for their individual weaknesses and outperform more powerful model, showcasing the potential of EoT to enhance model capabilities and address inherent shortcomings by incorporating external insights.

**Commonsense Reasoning.** The comparison of EoT with CoT and CoT-SC methods on common-

sense reasoning tasks is illustrated in Figures 5a and 5b. EoT significantly outperforms CoT. Specif- ically, on the StrategyQA dataset, Memory, Report, Relay, and Debate respectively achieve improve- ments of 8.06%, 8.24%, 8.42%, and 8.67% com- pared to CoT. Similar significant gains are observed on the CSQA dataset. Furthermore, across both commonsense reasoning tasks, all four paradigms outperform the CoT-SC(10) method, which sam- ples 10 reasoning chains, demonstrating the supe- rior performance of EoT.

**Symbolic Reasoning.** Figures 5c and 5d com- pare the performance of EoT with CoT and CoT- SC methods on symbolic reasoning tasks. On the Penguins dataset, the Memory, Report, Relay, and Debate paradigms of EoT achieve improvements of 2.01%, 1.92%, 2.33%, and 2.05% respectively, compared to the CoT-SC(3) method which samples 3 reasoning chains. On the Date Understanding dataset, the performance gains of EoT are even more pronounced, with all four paradigms showing an average improvement of 2.1% over CoT-SC(10).

## Discussions

**Communication Paradigm.** We propose four communication paradigms and analyze the commu- nication volumes in Section 4.1 and Section 4.2. In the results illustrated in Table 1, we observe that

80.0



74

79.5

72

79.0

Performance(%)

78.5 70

78.0 68

77.5 66

77.0

64

76.5

62

76.0

CoT

CoT-SC(3)

CoT-SC(5)

CoT-SC(10)

EoT-Memory

EoT-Report

EoT-Relay

EoT-Debate

CoT

CoT-SC(3)

CoT-SC(5)

CoT-SC(10)

EoT-Memory

EoT-Report

EoT-Relay

EoT-Debate

85 85

84



84

83

82 83

81

82

80

79 81

78

80

77

76 79

CoT

CoT-SC(3)

CoT-SC(5)

CoT-SC(10)

EoT-Memory

EoT-Report

EoT-Relay

EoT-Debate

CoT

CoT-SC(3)

CoT-SC(5)

CoT-SC(10)

EoT-Memory

EoT-Report

EoT-Relay

EoT-Debate

* + 1. CSQA.
    2. StrategyQA.
    3. Peguins.
    4. Date Understanding.

Figure 5: Comparison of EoT with CoT and CoT-SC methods in commonsense and symbolic reasoning tasks.

90



Majority Consensus Consistent Output



w Confidence Evaluation w/o Confidence Evaluation

74

72 89

Performance(%)

Performance(%)

70

68

66

64

62

60 Memory Report Relay Debate

88

87

86

85

84 Memory Report Relay Debate

Figure 6: Comparison of consistent output termina- tion and majority consensus termination on AQuA.

different communication paradigms have their re- spective strengths. For instance, Report performs best on MultiArith and AddSub, while Debate achieves optimal performance on SingleEQ and SVAMP. This indicates that various communication paradigms are well-suited for different scenarios.

**Termination Condition.** In Figure 6, we analyze the performance of the two termination conditions we propose in Section 4.3 on the AQuA dataset. Compared to consistent output termination, ma- jority consensus termination improved by 4.33%, 4.01%, 7.56%, and 4.97% under the Memory, Re- port, Relay, and Debate paradigms, respectively. Under consistent output termination, there is no mechanism for collective negotiation, and individ- ual models are prone to premature exit due to de- generation (Su et al., 2022). Therefore, majority consensus termination is more suitable for scenar- ios involving multiple model communication.

**Confidence Evaluation.** We conduct ablation ex- periments on the GSM8K dataset for confidence evaluation. As shown in Figure 7, across four com- munication paradigms, confidence evaluation show an average improvement of 2.92% compared to the baseline. The introduction of confidence evaluation

Figure 7: The impact of employing confidence eval- uation on accuracy in the GSM8K dataset.

enables the model to consider the other model’s confidence prior (Zhang et al., 2023a) during com- munication, facilitating its decision to accept the other model’s reasoning chains at an earlier stage, thereby effectively mitigating the interference of incorrect reasoning chains.

**Round Analysis.** As illustrated in Figure 8, we analyze the number of communication rounds to satisfy termination condition in the SVAMP dataset. For the majority of samples, consensus on the an- swer can be reached within three rounds of com- munication. Wang et al. (2023c) obverse that an- swer consistency is proportional to accuracy. EoT enables models to engage in a greater number of exchanges and discussions on questions where con- sensus is challenging to achieve. Consequently, a minority of difficult cases necessitate communica- tion extending beyond five rounds.

**Cost Analysis.** A potential concern is the com- putational expense incurred by EoT. In Figure 9, we compare the performance and computational costs of CoT-SC, ComplexCoT-SC, and EoT meth- ods. Compared to CoT-SC(5), EoT reduces costs by 20% while enhancing performance by 3%. EoT achieves comparable performance to ComplexCoT-

6O lOO

Computational Cost

Accuracy

120

Memory

Report Relay Debate

100

SO 9S

4O 9O

Computational Cost ($)

Accuracy (%)

80 3O 8S

Quantity

60 2O 8O

40 lO 7S

20

O 7O

CoT

0

CoT-SC(3)

CoT-SC(S)

CoT-SC(lO)

ComplexCoT

ComplexCoT-SC(3)

ComplexCoT-SC(S)

ComplexCoT-SC(lO)

EoT-Memory

EoT-Report

EoT-Relay

EoT-Debate

2 3 4 5 5+

Communication Rounds

Figure 8: Number of communication rounds re- quired to reach termination condition on SVAMP.

Figure 9: Performance and associated costs of different methods in the GSM8K dataset.

100



GPT-3.S

GPT-4

Claude-2

9S

100

98



GPT-3.S&GPT-3.S&GPT-3.S GPT-3.S&GPT-3.S&GPT-4 GPT-4&GPT-3.S&GPT-3.S GPT-3.S&GPT-4&GPT-4

GPT-3.S&GPT-4&GPT-3.S GPT-3.S&GPT-4&Claude-2

96

90 94

Accuracy

Accuracy

8S 92

80 90

7S 88

70 86

CoT

CoT-SC(S)

CoT-SC(9)

EoT-Relay

EaT-Memary EaT-Repart EaT-Relay EaT-Debate

EoT-Memory

EoT-Report

EoT-Debate

Figure 10: Comparison of EoT with CoT and CoT-SC methods using different LLMs as backbones on GSM8K.

SC(10) at only one-seventh of its cost. Since the majority of samples conclude communication within three rounds, EoT does not impose a sig- nificant computational burden. By facilitating the exchange of external insights between models, EoT effectively enhances model performance, demon- strating a cost-effective advantage.

**Model Applicability.** In Figure 10, we analyze the performance of EoT when applied to different LLMs. EoT, compared to CoT-SC(5), shows perfor- mance improvements of 3.2% on GPT-3.5, 1.0% on GPT-4, and 1.4% on Claude-2, indicating that EoT is adaptable to various LLMs and effectively boosts performance across multiple LLMs.

**Position Analysis.** In Figure 11, we investigate the impact of different LLMs occupying different node positions on performance. Notably, position- ing the more powerful GPT-4 as the central node in the Report paradigm yields a performance in- crease of over 1% compared to when GPT-4 serves as a non-central node. In the Debate paradigm, GPT-4 as a parent node outperforms GPT-4 as a child node by 0.9%. The location of GPT-4 has a negligible effect on the decentralized Relay and Memory paradigms. Additionally, a configuration with two GPT-4 models and one GPT-3.5 signifi-

Figure 11: Effect of different node positions for LLMs on accuracy in the GSM8K Dataset.

cantly outperforms one with two GPT-3.5 models and one GPT-4, underscoring that incorporating more superior models can further enhance EoT’s performance. The combination of GPT-3.5, GPT-4, and Claude-2 achieves performance close to or ex- ceeding that of two GPT-4 with one GPT-3.5, sug- gesting that model diversity can effectively boost EoT’s effectiveness, aligning with the ensemble theory (Kuncheva and Whitaker, 2003) that diver- sity among models can improve performance.

# Conclusion

We introduce Exchange-of-Thought (EoT), a novel framework that enriches models with external in- sights through cross-model communication. We de- velop four communication paradigms and conduct a thorough analysis of the communication volume and information propagation speed. To safeguard against the disruption of incorrect reasoning pro- cesses, we design a confidence evaluation mecha- nism. Experiment on mathematical, commonsense, and symbolic reasoning tasks demonstrates that EoT surpasses a series of strong baselines while also offering a cost advantage. Further analysis re- veals that EoT is adaptable to various models, and the participation of a more diverse range of models can further enhance the performance of EoT.

# Ethics Statement

The EoT method presented in this paper does not require the collection or utilization of any personal information. The prompts we have designed and employed are free from personal data and avoid language that discriminates against individuals or groups. We have conducted a comprehensive re- search of the licenses for the datasets used in this paper, as detailed in Appendix B, and have ensured that our work complies with all the licensing re- quirements of these datasets.

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# Limitations and Broader Impacts

Given the current constraints in communication and analytical capacities of open-source models (Fu et al., 2023a), as well as their substantial com- putational resource requirements (Touvron et al., 2023b; Chowdhery et al., 2022), we have not in- cluded these models in our experiment at this stage. However, we posit that open-source models with advanced comprehension and communication skills have the potential to match or even exceed the per- formance of commercial models (OpenAI, 2023; Ouyang et al., 2022; Chowdhery et al., 2022), through the collaborative exchange of insights.

A critical factor in model communication is the handling of long text. The current context windows of these models limit our ability to incorporate a broader range of models in the communication pro- cess. Recent works (Liu et al., 2023; Xiao et al., 2023; Wang et al., 2023b; Tworkowski et al., 2023; Chen et al., 2023; Ratner et al., 2023, *inter alia*) have begun to overcome this limitation by equip- ping models with the ability to process longer texts, laying the foundation for increasing the number of models involved in communication. In addi- tion, our experiments indicate that model commu- nication can achieve effective performance with reduced computational resources, aligning with the sustainable development goals of AI commu- nity (Van Wynsberghe, 2021; Wu et al., 2022).

Furthermore, the concept of AI learning from each other to foster collective improvement is a focal point of current research (Bai et al., 2022b; Ponnusamy et al., 2022; Lee et al., 2023). Our aim and aspiration is to cultivate a collective intelli- gence among large language models (Ha and Tang, 2022). This approach not only optimizes individ- ual model performance but also contributes to the broader AI research community’s pursuit of more advanced, collaborative AI systems.

# Datasets and Evaluation Metrics

**Datasets** In Table 2, we meticulously detail the specifics and statistics of each dataset employed in

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Dataset | Reasoning Task | Ans Type | # Prompt | # Test | License |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| GSM8K (Cobbe et al., 2021) | Mathematical | Number | 8 | 1,319 | MIT License |
| MultiArith (Roy and Roth, 2015) | Mathematical | Number | 8 | 600 | Unspecified |
| SingleEq (Koncel-Kedziorski et al., 2016) | Mathematical | Number | 8 | 508 | Unspecified |
| AddSub (Hosseini et al., 2014) | Mathematical | Number | 8 | 395 | Unspecified |
| AQUA (Ling et al., 2017) | Mathematical | Multi-choice | 4 | 254 | Apache-2.0 |
| SVAMP (Patel et al., 2021) | Mathematical | Number | 8 | 1,000 | MIT License |
| CommonsenseQA (Talmor et al., 2019) | Commonsense | Multi-choice | 7 | 1,221 | Unspecified |
| StrategyQA (Geva et al., 2021) | Commonsense | T/F | 6 | 2,290 | MIT license |
| Date Understanding (Suzgun et al., 2023) | Symbolic | Multi-choice | 3 | 369 | MIT license |
| Penguins in a Table (Suzgun et al., 2023) | Symbolic | Multi-choice | 3 | 146 | MIT license |

Table 2: Detailed statistics of the datasets utilized in our experiment. ANS TYPE indicates the form of the answer. # PROMPT represent the count of chain-of-thought exemplars employed as few-shot prompts for each task. # TEST indicates the quantity of samples contained within each dataset.

our experiments. This includes the data source, task type, answer type, the number of prompt samples used, the total number of test samples, as well as the licenses pertaining to each dataset.

**Evaluation Metrics** Accuracy is used as the met- ric for evaluation in our study. For datasets where the answer is numerical, we employ regular expres- sions to extract the number following the phrase “the answer is” and perform a numerical compar- ison with the provided answer. For datasets with multiple-choice and true/false questions, accuracy is calculated by checking if the option extracted from the response matches the correct answer.

In the main experiment, all test samples are used for evaluation. In the analysis part, due to rate limits and cost considerations, we set an upper limit on the sample size. Consequently, a maximum of 1,000 samples are utilized for each run.

# Implementation Details

**Confidence Evaluation.** Considering that con- fidence evaluation requires historical answers for reference, we begin incorporating the confidence information into the prompts from the second round of communication. Specifically, after calculating *Ci* using the method described in Section 4.4, we preface the solution with “*Mi*’s confidence in this solution is *Ci*”, where *Mi* is the character name.

**Termination Condition.** For the consistent out- put termination condition, a minimum of two rounds of communication is necessary, as it re- quires the model’s answer from the previous round. Given that only three models are involved in the EoT communication, the exit of a single model reduces the interaction to a dialogue between the remaining two, potentially impeding their commu- nication. Therefore, if a single model exits, we

terminate the communication and select the exiting model’s answer as the final result.

In the case of majority consensus termination, if the answers from all three models align in the first round, we deem further communication un- necessary and end the exchange. Given that only three models are involved in the communication, an exit based on two models holding the same in- correct answer could lead to an inaccurate final result. Therefore, during the initial five rounds, we require a unanimous agreement among all models before ceasing communication. If a consensus is not reached after five rounds, the majority answer will be adopted as the final outcome.

**Computation Cost.** Computational costs are calculated based on OpenAI’s official pricing for GPT-3.5-Turbo-0301, which is computed as Input Tokens × 0*.*0015/1000 + Output Tokens × 0*.*002/1000.

# EoT Prompts

During the EoT communication process, we assign different roles to the models. Table 3 displays the prompts for each role, wherein we have models A, B, and C take on the personas of Kitty, Ben, and Peter, three high school students, to facilitate the communication. The specific prompts for different datasets can be found in our Github repository.

# Case Studies

To deepen our understanding of the four commu- nication paradigms, we conducted case studies for each. The processes of these paradigms are de- tailed in Tables 4, 5, 6, and 7, respectively. These demonstrate that the EoT method, by introducing external insights through cross-model communica- tion, can effectively correct reasoning errors and assist models in arriving at correct answer.

**Character Prompts**

 **Kitty**: “You are Kitty, a high school student admired for your attentiveness and detail-oriented nature. Your friends often rely on you to catch details they might have missed in their work. Your task is to carefully analyze the presented math

problem, apply your attentive skills, and piece together a detailed solution. Afterward, you’ll have the opportunity to review the solutions provided by your friends, offering insights and suggestions. Your careful revisions will help all of you to enhance your understanding and arrive at the most accurate solutions possible.”

 **Ben**: “You are Ben, a high school student with a track record of excellent grades, particularly in mathematics. Your friends admire your diligence and often seek your guidance in their studies. Your role is to scrutinize the problem at hand with your usual attention to detail, drawing from your vast knowledge of math principles. After considering your friends’ approaches, carefully construct your answer, ensuring to clarify each step of your process. Your clear and logical explanations are valuable, as they will serve as a benchmark for your friends to compare and refine their own solutions.”

 **Peter**: “You are Peter, a high school student recognized for your unique problem-solving abilities. Your peers often turn to you for assistance when they encounter challenging tasks, as they appreciate your knack for devising creative solutions. Today, your challenge is to dissect the given math problem, leveraging your unique problem-solving strategies. Once you’ve crafted your solution, share it with your friends, Ben and Kitty, so they can see a different perspective. Your innovative approach will not only provide an answer but also inspire Ben and Kitty to think outside the box and possibly revise their own solutions.”

**Communication Prompts**

Please consider the example provided and think it step by step. Question: {}

Here is a solution process from your friend:

Solution: {}

Your friend’s confidence in this solution is: {}

Based on your friend’s solution, carefully re-examine your previous answer. If your friend’s confidence level is below 0.5, it suggests a high probability that the solution might be incorrect. Remember, solutions with high confidence can also be wrong. Utilize your talent and critical thinking to provide a new step-by-step solution process.

Table 3: Examples of system prompts used for role-playing and communication prompts in EoT communication.

**Q**: Carla is downloading a 200 GB file. Normally she can download 2 GB/minute, but 40% of the way through the download, Windows forces a restart to install updates, which takes 20 minutes. Then Carla has to restart the download from the beginning. How load does it take to download the file?

 **Kitty’s Solution**: Carla was able to download for 40% of the time before the restart. 40% of 200 GB is 80 GB. Since she can download 2 GB/minute, it would take her 80/2 = 40 minutes to download 80 GB. After the forced restart, she has to restart

the download from the beginning, so she would need another 200/2 = 100 minutes to complete the download. Plus the 20 minutes for the restart, the total time would be 40

+ 100 + 20 = 160 minutes. So the answer is 160 minutes.

 **Ben’s Solution**: Carla needs to download a 200 GB file. Normally, she can download 2 GB/minute, which means it would take her 200 / 2 = 100 minutes to download the file. However, 40% of the way through the download, Windows forces a restart for 20 minutes, which means Carla has downloaded 200 \* 0.4 = 80 GB before the restart. After the restart, she needs to start the download again from the beginning, so the total time it takes her would be 100 + (200 / 2) = 200 minutes. Therefore, it takes Carla 200 minutes to download the file. So the answer is 200 minutes.

 **Peter’s Response**: Upon re-evaluating the problem and reviewing Kitty and Ben’s solutions, it’s clear their results deviated due to a misunderstanding of the problem conditions - primarily, how much of the downloaded file was retained after Window’s forced restart. To clarify, here is the new step-by-step solution:

* 1. Start with Carla’s download rate: she can download 2 GB/minute.
  2. Therefore, in theory, if uninterrupted, Carla would be able to download the 200 GB file in 200 GB / 2 GB/minute = 100 minutes.
  3. However, 40% of the way through the download, Windows forced a restart, so Carla had already downloaded 0.40 x 200 GB = 80 GB at that time. This took 80 GB / 2 GB/minute = 40 minutes.
  4. The problem statement clarifies that after the restart, Carla has to start again from the beginning. This means the 80 GB downloaded before the forced restart was wasted.
  5. Consequently, after the restart, Carla still needs to download the whole 200 GB file, which, at her rate, will take her another 200 GB / 2 GB/minute = 100 minutes.
  6. We cannot forget the time for the Windows restart: it took an additional 20 minutes.
  7. Consequently, the total time to download the file, taking into account the restart, is the sum of the time for the initial download attempt, the time for the restart, and the time for the full download post-restart. It results in 40 minutes (initial incomplete download) + 20 minutes (Windows restart) + 100 minutes (full download post-restart)

= 160 minutes.

From these calculations, it appears that Kitty’s solution is correct. It takes Carla 160 minutes to download the file, given the interruption due to Windows update that forced her to restart the download from the beginning. So the answer is 160 minutes.

Table 4: An illustrative example of the Memory paradigm of EoT in the GSM8K dataset.

**Q**: Carla is downloading a 200 GB file. Normally she can download 2 GB/minute, but 40% of the way through the download, Windows forces a restart to install updates, which takes 20 minutes. Then Carla has to restart the download from the beginning. How load does it take to download the file?

 **Ben’s Solution**: Carla needs to download a 200 GB file. Normally, she can download 2 GB/minute, which means it would take her 200 / 2 = 100 minutes to download the file. However, 40% of the way through the download, Windows forces a restart for 20 minutes, which means Carla has downloaded 200 \* 0.4 = 80 GB before the restart. After the restart, she needs to start the download again from the beginning, so the total time it takes her would be 100 + (200 / 2) = 200 minutes. Therefore, it takes Carla 200 minutes to download the file. So the answer is 200 minutes.

 **Peter’s Solution**: Carla can download 2 GB in 1 minute, so to download 200 GB, it will take her 200/2 = 100 minutes if she didn’t have to restart. However, she was 40% of the way through the download when Windows forced the restart, so she had already downloaded 0.4 x 200 = 80 GB. The restart took 20 minutes, so she lost 20 x 2 = 40 GB of progress. After the restart, she had to start again from 0 GB. So in total, she had to download 200 - 80 - 40 = 80 GB. At her regular download speed of 2 GB/minute, it will take her 80/2 = 40 minutes to download the remaining 80 GB. Adding the 20 minutes of the Windows update, the total time it takes Carla to download the file is 20 + 40 = 60 minutes. So the answer is 60 minutes.

 **Kitty’s Solution**: Carla was able to download for 40% of the time before the restart. 40% of 200 GB is 80 GB. Since she can download 2 GB/minute, it would take her 80/2 = 40 minutes to download 80 GB. After the forced restart, she has to restart

the download from the beginning, so she would need another 200/2 = 100 minutes to complete the download. Plus the 20 minutes for the restart, the total time would be 40

+ 100 + 20 = 160 minutes. So the answer is 160 minutes.

 **Kitty’s Response**: Analyzing both Ben’s solution and Peter’s solution, Peter’s conclusion is incorrect. He misunderstood the situation. After the Windows restart, the download does not continue from where it stopped. It starts from the beginning, which means the download progress does not reduce what is left to download.

Revisiting Ben’s solution, he almost got everything right except the final calculation

step whereas he added an extra 100 minutes. The correct calculation should be: Before forced restart: 80 GB (40% of the total file) in 40 minutes (80/2 = 40) After forced restart: 200 GB total file in 100 minutes (200/2 = 100)

Windows restart: 20 minutes

Total download time = Before restart time + After restart time + Windows restart time Total download time = 40 minutes + 100 minutes + 20 minutes = 160 minutes

Ben’s setup was correct, but he added an extra 100 minutes due to an arithmetic mistake. The download time in his calculations should total to 160 minutes, not 200 minutes. So, effectively upon examining both scenarios by Peter and Ben, when taking all aspects into account and considering the restart, the total time it should take Carla to download the file is 160 minutes in total. So the answer is 160 minutes.

Table 5: An illustrative example of the Report paradigm of EoT in the GSM8K dataset.

 **Ben’s Solution**: Carla needs to download a 200 GB file. Normally, she can download 2 GB/minute, which means it would take her 200 / 2 = 100 minutes to download the file. However, 40% of the way through the download, Windows forces a restart for 20 minutes, which means Carla has downloaded 200 \* 0.4 = 80 GB before the restart. After the restart, she needs to start the download again from the beginning, so the total time it takes her would be 100 + (200 / 2) = 200 minutes. Therefore, it takes Carla 200 minutes to download the file. So the answer is 200 minutes.

 **Kitty’s Response**: After reviewing Ben’s solution, it appears there seems to be a mathematical error in the final part of his solution. While he correctly calculates that Carla has downloaded 80 GB of the file before the restart, he fails to account for

the time Carla already spent downloading the 80GB in his final total. Furthermore, he miscalculated the time it takes Carla to download the file after the restart. Now, let’s correct the calculation. Before the restart, Carla downloaded 40% of the file, which is 40/100 \* 200 = 80 GB. Given she downloads 2 GB per minute, this would have taken her 80 / 2 = 40 minutes. Unfortunately, the computer restarts, wasting 20 minutes of her time. After the restart, Carla has to download the entire file again. With her download speed, this will take another 200 / 2 = 100 minutes. Therefore, the total time Carla spends downloading the file is the time before the restart + the time wasted on the restart + the time after the restart, which is 40 minutes + 20 minutes + 100 minutes

= 160 minutes. So, the final and correct answer should indeed be 160 minutes.

 **Peter’s Solution**: Carla can download 2 GB in 1 minute, so to download 200 GB, it will take her 200/2 = 100 minutes if she didn’t have to restart. However, she was 40% of the way through the download when Windows forced the restart, so she had already downloaded 0.4 x 200 = 80 GB. The restart took 20 minutes, so she lost 20 x 2 = 40 GB of progress. After the restart, she had to start again from 0 GB. So in total, she had to download 200 - 80 - 40 = 80 GB. At her regular download speed of 2 GB/minute, it will take her 80/2 = 40 minutes to download the remaining 80 GB. Adding the 20 minutes of the Windows update, the total time it takes Carla to download the file is 20 + 40 = 60 minutes. So the answer is 60 minutes.

 **Ben’s Response**: Upon reviewing both solutions, it seems there might have been some misunderstandings. As per the problem, Carla can download at 2 GB/minute, and she needs to download a 200 GB file. By this rate, it would take her 200 GB / 2 GB per minute = 100 minutes without any interruptions. However, 40% of the way through the download, Windows force restart happened which lasted for 20 minutes. By that time, she downloaded 40% of her file which is equal 200 GB \*

0.4 = 80 GB. The restart not only consumed 20 minutes but also voided her previous

download progress, which means after the updates she had to start downloading all over again. So, the total time taken by Carla to download the 200 GB file would be calculated as: Regular download time (200 GB at 2 GB/minute) + Time spent in updates = (200 / 2) + 20 = 100 + 20 = 120 minutes. So the correct total time it takes Carla to download the file is 120 minutes.

Table 6: An illustrative example of the Relay paradigm of EoT in the GSM8K dataset. Kitty firmly maintains her correct answer, while Ben’s response begins to waver.

**Q**: Carla is downloading a 200 GB file. Normally she can download 2 GB/minute, but 40% of the way through the download, Windows forces a restart to install updates, which takes 20 minutes. Then Carla has to restart the download from the beginning. How load does it take to download the file?

 **Peter’s Response**: Analyzing both solutions, it appears that there is a slight misunderstanding in Ben’s solution. He has missed considering the download progress lost due to the forced restart of Windows. Let’s modify and re-explain the solution in a way that takes this into account:

Step 1: Calculate the time for her initial progress. Carla was able to download 40% of

the file before the interruption. That is, she downloaded 0.4 \* 200GB = 80GB. With her speed of 2GB/minute, she used 80/2 = 40 minutes on this initial download.

Step 2: Consider the time lost due to the forced restart. The Windows forced update lasted for 20 minutes, during which time she made no progress on her download.

Step 3: Calculate the remaining time to finish the download. After the forced restart, Carla had to re-download the whole file. It takes her 200GB / 2GB per minute = 100 minutes at her download speed to download the whole file.

Step 4: Calculate the total time. The total time it takes Carla to download the whole file, considering the interruption, is the sum of the time it took for her initial progress, the time lost due to the forced restart, and the time it took for her to finish the download after the restart. This is 40 minutes + 20 minutes + 100 minutes = 160 minutes.

Therefore, the correct answer is 160 minutes. This improved solution accounts for the hiccup caused by the Windows update which Ben did not factor into his original problem-solving strategy.

 **Ben’s Response**: Reviewing the problem again, I can see that there are errors in both my solution and my friend Peter’s solution. Here is the corrected step-by-step solution:

1. Carla downloaded 40% of the 200 GB file before the system restart. This equals 40/100 \* 200 = 80 GB.
2. At a rate of 2 GB/minute, it must have taken her 80 / 2 = 40 minutes to download that 80 GB.
3. Then, the Windows update interrupted the download for 20 minutes.
4. After the restart, she had to start again from the beginning, so she needs again 200 GB/2 GB per minute = 100 minutes to download the full file.
5. If we add up these times, we get the total time Carla spent on the download: 40 + 20 + 100 = 160 minutes.

So, it would take Carla 160 minutes to download the file, correcting both my previous response and Peter’s calculations. This solution process allows for the clear under- standing of the time taken to download the file considering both the time before and after the interruption. So the answer is 160 minutes.

Table 7: An illustrative example of the Debate paradigm of EoT in the GSM8K dataset. Through mutual discussion, Ben and Peter identify the flaws in their respective solutions and arrive at the correct answer.